A Comprehensive Mathematical Model for the Design of Cellular Manufacturing System

Published in 2006 in International Journal of Production Economics Vol. 103, 767 -783

A COMPREHENSIVE MATHEMATICAL MODEL FOR THE DESIGN OF CELLULAR MANUFACTURING SYSTEMS

Fantahun M. Defersha, Mingyuan Chen [1]

Department of Mechanical and Industrial Engineering, Concordia University 1455 Maisonneuve West, Montreal, Quebec, Canada H3G 1M8

Received: July 9, 2004; Revised: Nov.24, 2004, April 11, Aug.17, 2005; Accepted: Oct.30, 2005

Abstract: The design of cellular manufacturing systems involves many structural and operational issues. One of the important design steps is the formation of part families and machine cells. In this paper a comprehensive mathematical model for the design of cellular manufacturing systems based on tooling requirements of the parts and tooling available on the machines is proposed. The model incorporates dynamic cell configuration, alternative routings, lot splitting, sequence of operations, multiple units of identical machines, machine capacity, workload balancing among cells, operation cost, cost of subcontracting part processing, tool consumption cost, setup cost, cell size limits, and machine adjacency constraints. Numerical examples are presented to demonstrate the model and its potential benefits.

Keywords: Cellular Manufacturing, Dynamic Cell Configuration, Alternate Routings, Lot Splitting, Workload Balancing, Integer Programming

1. INTRODUCTION

A cellular manufacturing system (CMS) is a production approach aimed at increasing production efficiency and system flexibility by utilizing the process similarities of the parts. It involves grouping similar parts into part families and the corresponding machines into machine cells. This results in the organization of production systems into relatively self-contained and self-regulated groups of machines such that each group of machines undertake an efficient production of a family of parts. Such decomposition of the plant operations into subsystems may often lead to reduced paper work, reduced production lead time, reduced work-in-process, reduced labor, better supervisory control, reduced tooling, reduced setup time, reduced delivery time, reduced rework and scrap materials, and improved quality [Wemmerlöv and Johnson, 1997].

In the last three decades, research in CMS's has been extensive and literature in this area is abundant. Comprehensive summaries and taxonomies of studies devoted to part-machine grouping problems were presented by Wemmerlöv and Hyer [1986], Kusiak [1987], Selim, *et al* [1998], and Mansouri, *et al* [2000]. Methods for part family/machine cell formation can be classified as design-oriented or production-oriented. Design-oriented approaches group parts into families based on similar design features. An overview of design-oriented approaches based on classifi-

[1] For correspondence: mychen@me.concordia.ca, Tel: (514) 848-2424 Ext. 3134; Fax: (514) 848-3175

Table 1. List of Manufacturing Attributes

(1) Alternative Routing
  (a) Selecting the Best Route
  (b) Allowing Alternative Routing Coexist
(2) Demand Fluctuation
  (a) Deterministic
  (b) Probabilistic
(3) Dynamic Cell Reconfiguration
(4) Workload Balancing
  (a) Inter-cell Workload
  (b) Intra-cell Workload
(5) Lot-Splitting
(6) Types of Tools Required by a Part
(7) Types of Tools Available on a Machine
(8) Machine Proximity Constraint
  (a) Separation Constraint
  (b) Collocation Constraint
(9) Sequence of Operation
  (a) Used as input for determine magnitude of material flow
  (b) Used as Similarity measure between parts
(10) Setup Cost/Time
  (a) Setup Cost
  (b) Setup Time
(11) Cell / Part Family Size Constraint
  (a) Cell Size Constraint
  (b) Part Family Size Constraint
(12) Movement of Parts (Material Handling Cost)
  (a) Inter-Cell Movement
  (b) Intra-Cell Movement
(13) Facility layout
  (a) Inter-cell Layout
  (b) Intra-cell Layout
(14) Operator Allocation
(15) Machine Capacity
(16) Identical Machines
  (a) Within a Cell
  (b) In the Entire System
(17) Machine Investment Cost
(18) Subcontracting Cost
(19) Tool Consumption Cost
(20) Unit Operation Time
(21) Operation Cost

cation and coding was presented by Askin and Vakharia [1990]. Production-oriented techniques are for aggregating parts requiring similar processing. These approaches can be further classified into cluster analysis, graph partitioning, mathematical programming, Artificial Intelligence (AI) based approaches, and heuristics [Greene and Sadowski, 1984, Joines, *et al*, 1996].

Mathematical programming is widely used for modeling CMS problems. The objective of the mathematical programming model is often to maximize the total sum of similarities of parts in each cell, or to minimize inter-cell material handling cost. Purcheck [1974] applied linear programming techniques to a group technology problem. Kusiak [1987] proposed the generalized p-median model considering the presence of alternative routings. Shtub [1989] used the same approach and reformulated the problem as a generalized assignment problem. Wei and Gaither [1990] developed a 0-1 programming cell formation model to minimize bottleneck cost, maximize average cell utilization, and minimize intracell and inter-cell load imbalances. The bottleneck cost is related to the processing of bottleneck parts. Rajamani, *et al* [1990] proposed three integer programming models to consider budget and machine capacity constraints as well as alternative process plans. Askin and Chiu [1990] proposed a cost-based mathematical formulation and a heuristic graph partitioning procedure for cell formation. Shafer and Rogers [1991] applied a goal programming method to solving CMS problems for different system reconfiguration situations: setting up a new system and purchasing all new equipment, reorganizing the system using only existing equipment, and reorganizing the system using existing and some new equipment. Shafer, *et al* [1992] presented a mathematical programming model to address the issues related to exceptional elements. Heragu and Chen [1998] developed a mathematical programming model for cell formation and used Benders' decomposition to solve the problem.

Various methods have been proposed for cell formation incorporating several system features simultaneously. A list of these features is given in Table 1. A sample of 19 recently published articles and the corresponding system features considered in these articles are given in Table 2. The model presented in this paper provides a larger coverage of the attributes than the individual papers. A wider range of input data and cell formation criteria are incorporated than many of the models reviewed in Mansouri, *et al* [2000]. The rest of this paper is organized as follows. Detailed descriptions of the problem and the proposed model are given in Section 2. Numerical examples are presented in Section 3 to illustrate the proposed model. Discussion and conclusions are presented in Section 4.

2. THE MATHEMATICAL MODEL

2.1 Problem Description

Consider a manufacturing system consisting of a number of machines to process different parts. Each machine has a number of tools available on it and a part may require some or all of the tools on a given machine. A part may require several operations in a given sequence. An operation of a part can be processed by a machine if the required tool is available on that machine. If the tool is available on more than one machine type then the machines are considered as alternative routings for processing the part. An entire lot of a part may split

Table 2. Attributes Used in the Present Study and in a Sample of Recently Published Articles

Article/Attributes	1	1	2	2	3	4	4	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13	14	15	16	16	17	18	19	20	21	Total
	a	b	a	b		a	b				a	b	a	b	a	b	a	b	a	b	a	b			a	b						Attr.
Present Study (this paper)		×	×		×	×		×	×	×	×	×	×		×		×		×					×	×	×	×	×	×	×	×	21
Cao and Chen [2004]																	×		×					×	×	×				×		6
Jayaswal and Adil [2004]	×												×				×		×					×	×	×	×			×	×	10
Solimanpur, *et al* [2004]	×													×				×	×					×	×	×	×			×	×	10
Xambre and Vilarinho [2003]													×				×		×					×	×	×				×		7
Asokan, *et al* [2001]							×						×						×	×				×	×	×				×		8
Baykasoglu, *et al* [2001]	×					×			×	×				×			×	×	×					×	×	×				×		12
Diaz, *et al.* [2001]	×										×	×	×				×		×	×												7
Onwubolu and Mutingi [2001]							×										×		×													3
Akturk and Turkcan [2000]	×												×				×		×	×		×		×	×	×	×			×	×	12
Caux, *et al* [2000]	×												×				×		×					×	×	×				×		8
Mungwattana [2000]	×		×	×	×								×				×		×					×	×	×	×			×	×	13
Plaquin and Pierreval [2000]											×	×	×				×		×													5
Zhao and Zhiming [2000]							×						×				×		×	×				×	×	×				×		9
Sofianopoulou [1999]	×										×	×	×						×							×						6
Wicks and Reasor [1999]			×		×								×				×	×	×					×	×	×	×			×		11
Chen [1998]			×		×												×		×							×	×					6
Heragu and Chen [1998]		×									×	×					×		×					×	×	×	×			×		10
Selim, *et al* [1998]									×	×			×			×		×	×		×		×	×	×	×		×	×	×		14
Su and Hsu [1998]						×	×						×			×	×		×	×	×			×	×	×	×			×		13

Note: Attributes' names are referred in Table 1 into different cells for the processing of an operation. The manufacturing system is considered for a number of time periods. Each machine has a limited capacity expressed in hours during each time period. Machines can be duplicated to meet capacity requirements and to reduce or eliminate inter-cell movement. If additional machines are required in a given time period, the machines can be procured with certain limit. Assume that the demands for the part processing vary with time in a deterministic manner. Further assume that the processing, setup, and tool consumption costs do not depend on the planning period. Machines are to be grouped into relatively independent cells with minimum inter-cell movement of the parts. In grouping the machines, it is also required that the workload of the cells should be balanced. Machines that cannot be located in a same cell due to technical and environmental requirements should be separated. Machine pairs that utilize common resources are required to be located in the same cell. To address this multiple time period cell formation problem, a mixed integer programming model is formulated. The objective of the model is to minimize machine maintenance and overhead cost, machine procurement cost, inter-cell travel cost, machine operation and setup cost, tool consumption cost, and system re-configuration cost for the entire planning time horizon. The notations used in the model are presented below.

Indices:

Time period index: $t = 1, 2, ..., T$
Part type index: $i = 1, 2, ..., I$
Index of operations of part i: $j = 1, 2, ..., J_i$
Machine index: $k = 1, 2, ..., K$
Tool index: $g = 1, 2, ..., G$
Cell index: $l = 1, 2, ..., L$

Input Data:

- $d_i(t)$ Demand for part i in time period t
- V_i Unit cost to move part i between cells
- B_i Batch size of product i
- Φ_i Cost of subcontracting part i
- λ_{jig} Equals to 1, if operation j of part i requires tool g, 0 otherwise
- δ_{gk} Equals to 1, if tool g is available on machine k, 0 otherwise
- h_{jik} Processing time of operation j of part i on machine type k
- w_{jik} Tool consumption cost of operation j of each part i on machine type k
- μ_{jik} Setup cost for operation j of part i on machine type k
- $Q_k(t)$ Maximum number of machine type k that can be procured at the beginning of period t
- $P_k(t)$ Procurement cost of machine type k at the beginning of period t
- H_k Maintenance and overhead costs of machine type k per time period t
- O_k Operation cost per hour of machine type k
- C_k Capacity of one machine of type k for one time period
- LB_l Minimum number of machines in cell l
- UB_l Maximum number of machines in cell l
- I_k^+ Cost of installing one machine of type k
- I_k^- Cost of removing one machine of type k
- q $0 \le q < 1$; A factor for the work load of a cell being as low as $q \times 100\%$ from the average work load per cell
- $Z_i(t)$ The number of cells among which an entire lot of part i may split into during time period t for the processing of certain operations; $Z_i(t)] \in \{1,\ 2, ...,\ L\}$
- S A set of machine pairs $\{(k^a, k^b)/k^a, k^b \in \{1, \cdots, K\}, k^a \neq k^b$, and k^a cannot be placed in the same cell with $k^b\}$
- Ω A set of machine pairs $\{(k^c, k^d)/k^c, k^d \in \{1, \cdots, K\}, k^c \neq k^d$, and k^c should be placed in the same cell with $k^d\}$
- M A large positive number

Decision Variables:

General Integer:

- $N_{kl}(t)$ Number of type k machines to assign to cell l at the beginning of period t
- $y_{kl}^+(t)$ Number of type k machines to add to cell l at the beginning of period t
- $y_{kl}^-(t)$ Number of type k machines to remove from cell l at the beginning of period t

Continuous:

- $\eta_{jikl}(t)$ The proportion of the total demand of part i with the j^{th} operation to perform by machine type k in cell l during period t
- $\bar{\eta}_i(t)$ The proportion of the total demand of part i to be subcontracted in time period t

Auxiliary Binary Variables:

The auxiliary binary variables are used to formulate logical constraints. The values of these variables are not required to make decisions for system configuration and operation assignments. These variable are:

- $r_{kl}(t)$ Equals to 1, if type k machines are to be assigned to cell l during time period t; 0, otherwise
- $p_{jil}(t)$ Equals to 1, if operation j of part i is to be processed in cell l during period t; 0, otherwise

2.2 Objective Function and Constraints

The mixed integer programming model for the CMS design is presented below.

Objective:

$$\text{Minimize } \; Z = \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{k=1}^{K} N_{kl}(t)\cdot H_k$$
$$+\sum_{t=1}^{T}\sum_{k=1}^{K} P_k(t)\cdot max\left\{0, \; \left(\sum_{l=1}^{L} N_{kl}(t) - \sum_{l=1}^{L} N_{kl}(t-1)\right)\right\}$$
$$+\frac{1}{2}\sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J_i-1}\left(d_i(t)\cdot V_i \left|\sum_{k=1}^{K}\eta_{j+1,ikl}(t) - \sum_{k=1}^{K}\eta_{jikl}(t)\right|\right)$$
$$+\sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J_i} d_i(t)\cdot\eta_{jikl}(t)\cdot h_{jik}(t)\cdot O_k$$
$$+\sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J_i} d_i(t)\cdot\eta_{jikl}(t)\cdot w_{jik}$$
$$+\sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J_i} \frac{d_i(t)\cdot\eta_{jikl}(t)}{B_i}\cdot\mu_{jik}$$
$$+\sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{k=1}^{K}\left(I_k^+\cdot y_{kl}^+(t) + I_k^-\cdot y_{kl}^-(t)\right)$$
$$\sum_{t=1}^{T}\sum_{i=1}^{I}\Phi_i\cdot d_i(t)\cdot\bar{\eta}_i(t) \quad (1)$$

Subject to:

$$d_i(t)\cdot\sum_{l=1}^{L}\sum_{k=1}^{K}\eta_{jikl}(t) = d_i(t)(1-\bar{\eta}_i(t)); \quad \forall(i,j,t) \quad (2)$$

$$\eta_{jikl}(t) \le \lambda_{jig}\times\delta_{gk}; \forall(i,j,k,l,t,g) \quad (3)$$

$$\sum_{k=1}^{K}\eta_{jikl}(t) \le p_{jil}(t); \forall(i,j,l,t) \quad (4)$$

$$\sum_{l=1}^{L} p_{jil}(t) \le Z_i(t); \forall(i,j,t) \quad (5)$$

$$C_k\cdot N_{kl}(t) \ge \sum_{i=1}^{I}\sum_{j=1}^{J_i} d_i(t)\cdot\eta_{jikl}(t)\cdot h_{jik}; \quad \forall(k,l,t) \quad (6)$$

$$\sum_{l=1}^{L} N_{kl}(t) - \sum_{l=1}^{L} N_{kl}(t-1) \le Q_k(t); \quad \forall(k,t) \quad (7)$$

$$\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J_i} d_i(t)\cdot\eta_{jikl}(t)\cdot h_{jik} \ge \frac{q}{L}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J_i} d_i(t)\cdot\eta_{jikl}(t)\cdot h_{jik}; \quad \forall(l,t) \quad (8)$$

$$LB_l \le \sum_{k=1}^{K} N_{kl}(t) \le UB_l; \forall(l,t) \quad (9)$$

$$N_{kl}(t) = N_{kl}(t-1) + y_{kl}^+(t) - y_{kl}^-(t); \quad \forall(k,l,t) \quad (10)$$

$$N_{kl}(t) \le M\cdot r_{kl}(t); \forall(k,l,t) \quad (11)$$

$$r_{kl}(t) \le N_{kl}(t); \forall(k,l,t) \quad (12)$$

$$r_{k^a l}(t) + r_{k^b l}(t) \le 1; (k^a,k^b)\in S, \forall(l,t) \quad (13)$$

$$r_{k^c l}(t) - r_{k^d l}(t) = 0; (k^c,k^d)\in \Omega, \forall(l,t) \quad (14)$$

$$0 \le \bar{\eta}_i(t) \le 1; \forall(i,t) \quad (15)$$

$$y_{kl}^+(t), \; y_{kl}^-(t), \; N_{kl}(t) \in \{0,1,2,\cdots\} \; \& \; p_{jil}(t), \; r_{kl}(t)\in\{0,1\}; \forall(i,j,k,l,t) \quad (16)$$

Objective Function: The 1st term of Z is machine maintenance and overhead costs. The 2nd term is machine procurement cost where $\sum_{l=1}^{L} N_{kl}(t), \; \forall t \ge 1$, is the number of machines of type k in the system at the beginning of period t. $\sum_{l=1}^{L} N_{kl}(0)$ is the number of machines of type k available from a previous system if the problem is to reconfiguring an existing system. For setting up a new system, $\sum_{l=1}^{L} N_{kl}(0) = 0, \; \forall k$. The 3rd term represents the inter-cell material handling cost. Assume that the costs of moving the same material between different cells are the same since the fixed costs involved in moving materials are normally large while the distance related cost components are typically small [Heragu and Chen, 1998], and hence are negligible. The 4th-7th terms of Z are machine operating cost, tool consumption cost, setup cost, and machine relocation cost, respectively. The 8th term is the cost for subcontracting parts.

Model Constraints: Eq. (2) is to ensure that if a part is not subcontracted, the processing of each operation of this part must be assigned to a machine. An assignment of an operation of a part is permitted only to a machine having the required tool using (3). This constraint is also for limiting the values of $\eta_{jikl}(t)$ within $[0,1]$. The processing of an operation j of part i is allowed to be performed in at most $Z_i(t)$ cells in time period t with (4) and (5). Machine capacity constraints are in (6). Eq. (7) limits the number of type k machines to procure at the beginning of period t to maximum possible. Workload balancing among cells is enforced with (8) where the factor $q\in[0,1)$ is used to determine the extent of the workload balance. If the number of cells is L, the minimum allowable workload of a cell is $\frac{q}{L}\times 100\%$ of the total workload in terms of processing time. The maximum allowable workload is given by $\left(\frac{q}{L} + 1 - q\right) \times$ 100% of the total workload. If q is chosen close to 1.0, the allowable workload of each cell will be close to the average workload given by $\frac{1}{L} \times 100\%$ of the total workload. Lower and upper bounds on the sizes of the cells are enforced with (9). Eq. (10) is to ensure that the number of machines of type k in the current period in a particular cell is equal to the number of machines in the previous period, adding the number of machines moved in and subtracting the number of machines moved out of the cell. Eqs. (11) and (12) are for setting $r_{kl}(t)$ to 1 if at least one type k machine is located in cell l during period t, 0 otherwise. Eq. (13) is to ensure that machine pairs included in S should not be placed in the same cell. Eq. (14) is to ensure that machine pairs included in Ω should be placed in the same cell. The values of $\bar{\eta}_i(t)$ are limited within $[0, 1]$ by (15).

2.3 Features of the Model

The distinguishing feature of the model is that it is for simultaneously addressing several pragmatic issues in the design of a CMS.

Dynamic Reconfiguration of Cells: In the presence of product mix variations, cell reconfiguration is a promising strategy to consider so that the manufacturing system may remain efficient. With increased demand for manufacturing flexibility, this strategy becomes more prominent in designing manufacturing cells [Chen, 1998]. As stated in a US National Research Council document [National Research Council, 1998], reconfigurable manufacturing is considered by many manufacturing experts as one of the most important technologies in advanced manufacturing systems. Designing a CMS in a dynamic environment was also discussed in Mungwattana [2000], Seifoddini [1990], Wicks and Reasor [1999], Chen [1998], and Harhallakis, *et al.* [1994]. However, manufacturing system reconfigurations may be attainable in certain light industries (such as electronic industries) or in a system with machine tools and equipment specially designed to make system reconfiguration practical. A virtual CMS (VCMS) will be a better approach for system analysis when physically moving machines around is too expensive or practically impossible. The reader is referred to Ko and Egbelu [2003] and Saad [2003] for recent development in VCMS research.

Alternative Routings: The presence of alternative routings is typical in many discrete, multi-batch, small lot size production environments. Routing flexibility increases the number of ways to form manufacturing cells. The mathematical model was formulated based on the tooling requirements of the parts and tooling availability of the machines. If a tool is available on more than one machine, then these machines are considered as alternative routings for operations requiring that particular tool. Researchers who considered dynamic cell reconfiguration have either ignored routing flexibility (e.g., Chen [1998]) or only chosen one route for each part from the available routes and do not suggest alternative routes to coexist (e.g., Wicks and Reasor [1999] and Mungwattana [2000]). Ignoring the remaining alternative routings may result in an increased operation cost and additional investment in machines. In the proposed model, alternative routings are considered and allowed to coexist and share the total production volume if an economic advantage can occur.

Lot Splitting: Lot splitting is a process used primarily in batch manufacturing scheduling. It is for dividing large orders into smaller batches providing the opportunity for simultaneous processing of orders to more than one work center. This may result in reduced flow time [Jacobs and Bragg, 1996] and better due date performance [Wagner and Ragatz, 1994]. In the context of CMS operation, Lockwood, *et al* [2000] and Süer, *et al* [1999] used the concept of lot splitting to improve the effectiveness of scheduling decisions. We introduced lot splitting at the design phase of a CMS because it may result in improved machine utilization, reduced inter-cell movement, decreased operation cost, reduced machine investment, and evenly distributed workload.

Sequence of Operations: Despite a large number of published papers on cell formation, very few authors have considered operation sequence in calculating inter-cell material movement [Jayaswal and Adil, 2004]. Cell formation methods, without using operation sequence data, may calculate inter-cell movement based on the number of cells that a part will visit in the manufacturing process. However, the number of cells visited by the part can be less than the actual number of inter-cell movements since the part may travel back and forth between cells. Such movements may not be accurately reflected without properly using operation sequence data. In the proposed model, sequence data are explicitly used to obtain accurate counts of the inter-cell movements of the parts.

Workload Balancing: Workload balancing contributes to a smooth running of the system and better performance in terms of throughput, makespan, flow time, and tardiness [Kim, 1993]. Balancing workload reduces work-in-process inventory, improves material flow through the system, and prevents heavy utilization of some cells and lower utilization of others [Baykasoglu, *et al*, 2001]. In this paper, the formulated model enables the system designer to set the level of workload balancing among the cells.

Machine Adjacency Requirement: A number of authors addressed machine adjacency requirement in CMS design [Diaz, *et al.*, 2001, Plaquin and Pierreval, 2000, Sofianopoulou, 1999, Heragu and Chen, 1998]. Such requirements exist since some machines must be separated from each other while other machines must be placed together due to technical and safety considerations. For example, machines that produce vibrations, dust, noise, or high temperatures may need to be separated from electronic assembly and final testing. In other situations, certain machines should be placed in the same cells. For example, a heat treatment station and a forging station may be placed adjacent to each other for safety reasons. Machines that share a common resource or those that require a particular operator's skill may also be placed in a same cell.

2.4 Linearizing the Objective Function

The objective function in the model is a non-linear function due to the *max* function and the absolute values in the second and third cost elements. These terms can be linearized using the procedures given below.

Linearizing the *max* Function: The *max* function in the second cost term can be linearized by introducing non-negative real variables $f_k^+(t)$ and $f_k^-(t)$, and a binary variable $\xi_k(t)$. The term $max\left\{0,\ \left(\sum_{l=1}^{L} N_{kl}(t) -\ \sum_{l=1}^{L} N_{kl}(t-1)\right)\right\}$ can then be replaced by $f_k^+(t)$ with the following added constraints:

$$\left(\sum_{l=1}^{L} N_{kl}(t) - \sum_{l=1}^{L} N_{kl}(t-1)\right) = f_k^+(t) - f_k^-(t); \forall (k,t) \qquad (17)$$

$$f_k^+(t) \leq M \cdot \xi_k(t); \forall (k,t) \qquad (18)$$

$$f_k^-(t) \leq M \cdot (1 - \xi_k(t)); \forall (k,t) \qquad (19)$$

$$\xi_k(t) \in \{0,1\}; \forall (k,t) \qquad (20)$$

Linearizing the Absolute Value Term: The absolute value $\left|\sum_{k=1}^{K} \eta_{j+1,ikl}(t) -\ \sum_{k=1}^{K} \eta_{jikl}(t)\right|$ in the third cost element of the objective function can be linearized by introducing non-negative real variables $n_{ijl}^+(t)$ and $n_{ijl}^-(t)$, and a binary variable $\beta_{ijl}(t)$. The term then can be replaced by $n_{ijl}^+(t) + n_{ijl}^-(t)$ with the following added constraints:

$$\sum_{k=1}^{K} \eta_{j+1,ikl}(t) = \sum_{k=1}^{K} \eta_{jikl}(t) + n_{ijl}^+(t) - n_{ijl}^-(t); \forall (i,j,l,t) \qquad (21)$$

$$n_{ijl}^+(t) \leq M \cdot \beta_{ijl}(t); \forall (i,j,l,t) \qquad (22)$$

$$n_{ijl}^-(t) \leq M \cdot (1 - \beta_{ijl}(t)); \forall (i,j,l,t) \qquad (23)$$

$$\beta_{ijl}(t) \in \{0,1\}; \forall (i,j,l,t) \qquad (24)$$

After these two terms are linearized, the objective function of the integer programming model has linear terms only. All constraints in the model are also linear. The number of variables and number of constraints in the linearized model are presented in Tables 3 and 4, respectively, based on the variable indices.

3. NUMERICAL EXAMPLES

Several example problems, all solved with LINGO, a commercially available optimization software, are presented in this section. Example 1 is explained in detail for its input data and computational results. Since other example problems are similar to Example 1, only summarized results are presented to further illustrate the CMS design issues addressed with the proposed model.

3.1 Example 1

In solving this example, we consider 10 different types of machines, 25 part types, and two planning time periods. The machines are to be grouped into three relatively independent cells and reconfiguration is to be performed at the beginning of the second period to respond to the changes of production demand.

3.1.1. Input Data and Problem Size The input data of this example are given in Tables 5 to 10. In Table 5, data on production batch size, unit cost of inter-cell movement, and the demand for the parts in the two time periods are given. The tool requirements for the various operations of the parts are given in Table 6. In Table 7, data of tool availability on different machines are presented. Table 8 contains the data related to alternative routings generated from matching tool requirement by operations of the parts and tool availability on the machines. In Table 9 is the data for machine overhead costs, operating costs, machine installation and removal costs, machine capacities, the maximum number of machines that can be procured, and machine procurement costs. For the numerical examples, we assume that machine installation cost is the same as machine removal cost. Data for the number of cells to be formed, lower and upper bounds for the cell sizes, list of machine pairs that cannot be placed in a same cell and work load balancing

Table 3. Number of Variables in the Linearized Model

Variable Name	Nature of Variable	Variable Count	Variable Name	Nature of Variable	Variable Count
$\eta_{jikl}(t)$	Continuous	$K \times L \times T \times OP$	$y^+_{kl}(t)$	Gen. Integer	$K \times L \times T$
$\eta_i(t)$	Continuous	$I \times T$	$y^-_{kl}(t)$	Gen. Integer	$K \times L \times T$
$f^+_k(t)$	Continuous	$K \times T$	$r_{kl}(t)$	Binary	$K \times L \times T$
$f^-_k(t)$	Continuous	$K \times T$	$p_{jil}(t)$	Binary	$L \times T \times OP$
$n^+_{jil}(t)$	Continuous	$L \times T \times OP$	$\xi_k(t))$	Binary	$K \times T$
$n^-_{jil}(t)$	Continuous	$L \times T \times OP$	$\beta_{ijl}(t)$	Binary	$L \times T \times OP$
$N_{kl}(t)$	Gen. Integer	$K \times L \times T$			

OP: Total number of operations in all of the parts

Table 4. Number of Constraints in the Linearized Model

Eq. No.	Total Count	Eq. No.	Total Count	Eq. No.	Total Count
2	$T \times OP$	9	$L \times T$	17	$K \times T$
3	$K \times L \times T \times OP$	10	$K \times L \times T$	18	$K \times T$
4	$L \times T \times OP$	11	$K \times L \times T$	19	$K \times T$
5	$T \times OP$	12	$K \times L \times T$	21	$L \times T \times OP$
6	$K \times L \times T$	13	$N(S) \times L \times T$	22	$L \times T \times OP$
7	$K \times T$	14	$N(\Omega) \times L \times T$	23	$L \times T \times OP$
8	$L \times T$	15	$I \times T$		

$N(S)$: Number of machine pairs in S
$N(\Omega)$: Number of machine pairs in Ω

factor q are given in Table 10. To setup a new system at the beginning of the first time period, we set $\sum_k N_{kl}(0) = 0, \ \forall l$. We assume that no part processing will be contracted out, so the subcontracting cost Φ_i was given a large number for each part type. Since the number of parts in the numerical example is small, the differences in the tool consumption cost for a given tool type among the various alternative routes are negligible and should not influence the choice of the alternative routes. Hence, tool consumption costs were not considered in this small example problem.

With the data and assumptions, the linearized model has 16,930 variables including 5,320 integer variables. The corresponding number of constraints is 24,030. These counts of variables and constraints can be reduced by removing variables that can be fixed to zero from the model. The variables which can be fixed to zero were removed from the model using sparse set membership filtering technique of LINGO [Lindo Systems Inc., 2002]. After these variables are fixed, some of the constraints became redundant and were subsequently removed. The resulting formulation has a total of only 6,437 variables and 2,120 are integer variables. The number of the corresponding constraints was reduced to 5,602.

3.1.2. Solution of Example 1 The cells generated during each time period and the part assignment to the various cells are given in Tables 11 and 12. Since the full listing of the values of all of the variables $\eta_{jikl}(t)$ may not be useful, we only present values of a sample of these variables for part types 1, 6, 10, 15 and 21 in Table 13. Part 15 is entirely processed in cell 1. This is indicated in Table 11 by a unit value corresponding to the machines required to process this part. Many other parts are also processed without inter-cell movements. Similar to part 15, part 1 is also processed in one cell during period 1. Notice that the fourth operation of part 1 is performed partially by machine type 3 and partially by machine type 4 due to alternative routings that coexist. Part 10 is processed partially in cell 1 and partially in cell 2 due to lot splitting. One can see that part 10 appears in columns 7 and 10 and there are no values outside the diagonal block corresponding to this part. Similar to part 10, part 6 is also processed in two cells during period 2. There is a combined effect of lot splitting and alternative routings as the fourth and fifth operations of this part are performed by machine type 1 in cell 2 and by machine type 2 in cell 3. The first four operations of part 21 are processed partially in cell 2 and partially in cell 3. The fifth operation is processed partially in cell 3 and partially in cell 1 while the last three operations are performed within cell 1 only. Hence, there is an inter-cell movement from cell 2 and cell 3 to cell 1. This is reflected by elements outside the diagonal block in Table 12. The reconfiguration performed at the beginning of period 2 can be found from the data given in Tables 11 and 12. For example, six machines of type 2, one machine of type 3, and four machines of type 6 are added to cell 1 in period 2. At the same time, one unit of machine types 1, 4, 7, 8, 9, and 10 as well as two units of machine type 5 are removed from cell 1 at the beginning of the second time period.

3.1.3. Solution Analysis for Example 1 **Workload Balancing:** For the generated three cells, the minimum allowable work load is $\frac{q}{L} \times 100 =$

Table 5. Data for the Part Types

Part No.	Batch Size B_i	Cost of Inter-Cellular Movement per Unit V_i	Demand During Period t $t=1$	$t=2$
1	100	6	4000	0
2	150	12	0	3200
3	300	27	4000	2500
4	200	18	0	4500
5	100	15	4400	0
6	100	15	0	4500
7	150	24	0	4500
8	150	12	3600	0
9	150	12	3400	0
10	200	18	6500	0
11	300	24	0	4500
12	100	12	0	3550
13	100	27	2000	6000
14	150	21	4000	0
15	300	27	4400	4500
16	150	24	0	6000
17	200	12	3500	0
18	100	21	3800	5700
19	100	18	4800	0
20	100	12	0	3800
21	100	24	0	3000
22	150	15	4700	3000
23	120	18	5400	0
24	150	27	0	4500
25	150	18	0	4500

Table 6. Tool Requirement of Parts

Part No.	The Index g of the Tool Required for Processing Operation j $j=1$	2	3	4	5	6	7	8	9
1	1	10	3	13	15				
2	30	31	35	25	27	26	37	36	38
3	6	7	8	2	1	22	24	23	
4	16	17	21	36	38				
5	34	37	38	26	28	29	27	32	33
6	17	19	21	1	2	37			
7	1	2	3	11	12	15	4	5	20
8	6	7	22	23	8	9			
9	28	29	33	38	39	34	35		
10	16	17	18	20	39	40			
11	1	10	12	3	6	24			
12	8	9	1	25	24				
13	27	26	25	30	31	33	19	18	20
14	22	23	24	17	21				
15	3	11	12	5	15	35			
16	34	35	25	26	29				
17	2	8	9	24					
18	10	11	13	1	15				
19	18	19	20	36	38	37			
20	6	7	8	1	2	9	23	24	
21	39	40	27	28	29	22	30	31	
22	1	3	10	12	6	24			
23	1	2	17	19	21	36			
24	22	23	7	6	9				
25	28	29	32	34	35	39			

Table 7. Tool Availability on Machines

Machine Type k	Indices of the Available Tools	Machine type k	Indices of the available tools
1	1, 2, 3, 4, 5	6	22, 23, 24
2	1, 2, 6, 7, 8, 9	7	25, 26, 27, 28, 29
3	10, 11, 12, 13	8	30, 31, 32, 33, 34, 35
4	12, 13, 14, 15	9	36, 37, 38, 39, 40
5	16, 17, 18, 19, 20, 21	10	36, 37, 38

Table 8. Routes and Alternative Routings

Part No.	Operation No. 1	2	3	4	5	6	7	8	9
1	(1,100,12) (2,110,11)	(3,80,18)	(1,150,10)	(3,120,10) (4,155,8)	(4,90,16)				
2	(8,40,16)	(8,40,14)	(8,40,16)	(7,90,12)	(7,120,12)	(7,90,10)	(9,100,10) (10,120,10)	(9,130,17) (10,120,18)	(9,180,16) (10,120,18)
3	(2,90,5	(2,90,10)	(2,90,15)	(1,100,10) (2,120,9)	(1,100,16) (2,120,15)	(6,30,20)	(6,40,15)	(6,40,10)	
4	(5,30,8)	(5,30,10)	(5,90,12)	(9,70,16) (10,140 15)	(9,70,20) (10,120,19)				
5	(8,60,9)	(9,90,10) (10,120,9)	(9,60,12) (10,120,11)	(7,90,6)	(7,120,8)	(7,100,8)	(7,120,10)	(8,40,16)	(8,60,12)
6	(5,30,16)	(5,60,10)	(5,90,8)	(1,100,12) (2,100,12)	(1,100,16) (1,120,14)	(9,100,12) (10,120,12)			
7	(1,100,5) (2,120,5)	(1,100,8) (2,120,9)	(1,150,6)	(3,80,10)	(3,120,12) (4,140,11)	(4,90,14)	(1,150,16)	(1,150,10)	(5,70,8)
8	(2,90,6)	(2,90,8)	(6,30,12)	(6,40,14)	(2,110,8)	(2,110,8)			
9	(7,120,6)	(7,100,12)	(8,60,16)	(9,60,14) (10,120,13)	(9,80,20)	(8,60,6)	(8,60,16)		
10	(5,30,12)	(5,30,6)	(5,60,8)	(5,70,16)	(9,80,7)	(9,80,4)			
11	(1,100,7) (2,90,7)	(3,80,10)	(3,120,10) (4,140,9)	(1,150,15)	(2,90,8)	(6,40,5)			
12	(2,110,7)	(2,110,11)	(1,100,8) (2,120,7)	(7,90,15)	(6,40,15)				
13	(7,20,5)	(7,90,14)	(7,90,8)	(8,40,10)	(8,40,6)	(8,60,5)	(5,60,10)	(5,60,12)	(5,70,12)
14	(6,30,16)	(6,40,8)	(6,40,12)	(5,30,6)	(5,90,4)				
15	(1,150,5)	(3,80,10)	(3,120,12) (4,140,11)	(1,150,5)	(4,90,8)	(8,60,4)			
16	(8,60,12)	(8,60,10)	(7,90,14)	(7,90,9)	(7,100,14)				
17	(1,100,16) (2,120,16)	(2,110,6)	(2,110,18)	(6,40,12)					
18	(3,80,5)	(3,80,8)	(3,130,10) (4,140,9)	(1,100,8) (2,120,7)	(4,90,16)				
19	(5,60,10)	(5,60,9)	(5,70,13)	(9,70,12) (10,120,11)	(9,60,12) (10,110,11)	(9,70,16) (10,120,15)			
20	(2,90,8)	(2,90,6)	(2,110,6)	(1,100,9) (2,100,9)	(1,100,16) (2,110,15)	(2,110,7)	(6,40,15)	(6,40,12)	
21	(9,80,14)	(9,80,10)	(7,120,8)	(7,120,5)	(7,100,5)	(6,30,12)	(8,40,18)	(8,40,12)	
22	(1,100,18) (2,105,17)	(1,150,8)	(3,80,14)	(3,140,16) (4,125,15)	(2,90,12)	(6,40,12)			
23	(1,100,8) (2,120,7)	(1,110,20) (2,140,18)	(5,30,7)	(5,60,9)	(5,90,12)	(9,70,12) (10,125,11)			
24	(6,30,14)	(6,40,16)	(2,90,8)	(2,90,10)	(2,110,8)				
25	(7,120,8)	(7,120,6)	(8,40,12)	(8,60,10	(8,60,6)	(9,80,5)			

Routes and alternative routings were determined by matching the tool requirement of the parts and the tool availability on the machines.

Table 9. Data for Machines

Machine Type k	H_k	O_k	C_k	I_k^+	I_k^-	$Q_k(t)$ $t=1$	$t=2$	$P_k(t)$ $t=1$	$t=2$
1	500.00	12.00	2000	75.00	75.00	7	1	12500.00	12700.00
2	600.00	11.00	1800	100.00	100.00	12	1	11800.00	12200.00
3	800.00	8.00	1800	140.00	140.00	10	3	10000.00	10200.00
4	400.00	10.50	2000	90.00	90.00	7	2	11200.00	11200.00
5	900.00	8.50	2000	80.00	80.00	8	4	17200.00	17200.00
6	450.00	10.00	2200	100.00	100.00	9	1	13200.00	13200.00
7	650.00	9.00	1840	70.00	70.00	10	2	16200.00	16200.00
8	450.00	8.00	1800	70.00	70.00	10	1	15000.00	15000.00
9	650.00	13.00	2200	80.00	80.00	10	2	12200.00	12200.00
10	300.00	12.00	2200	75.00	75.00	10	4	13200.00	13200.00

$\frac{0.9}{3} \times 100 = 30\%$ of the total workload in processing time with the maximum being $\left(\frac{q}{L} + 1 - q\right) \times 100 = \left(\frac{0.9}{3} + 1 - 0.9\right) \times 100 = 40\%$. In order to see the impact of enforcing workload balancing, we recalculated the example problem without this

Table 10. Miscellaneous Data

Number of cells	3
Lower bound for the cell size	2 machines
Upper bound for the cell size	25 machines
Pair of machines that should not be located in the same cell (arbitrarily selected)	{2, 4} and {6, 9}
Pair of machines that should be located in the same cell (arbitrarily selected)	{1, 3}
Work load balancing factor, q	0.90

Table 11. Part-Cell Assignment for Period 1

	Machine		Parts Types																
Cell	Type	Qnt.	1	5	9*	10*	23	9*	10*	13*	15	18	19	3	8	13*	14	17	22
C1	M1	2	1				1												
	M3	1	1																
	M4	1	1																
	M5	2				0.33	1												
	M7	2		1	0.41														
	M8	2		1	0.41														
	M9	1		1	0.41	0.33	1												
	M10	1		1	0.41		1												
C1	M1	1									1	1							
	M3	1									1	1							
	M4	2									1	1							
	M5	3							0.67	0.31			1						
	M7	2						0.59		0.31									
	M8	1						0.59		0.31	1								
	M9	1						0.59	0.67				1						
	M10	2						0.59					1						
C3	M1	1																	1
	M2	6												1	1			1	1
	M3	2																	1
	M5	1														0.69	1		
	M6	4												1	1		1	1	1
	M7	1														0.69			
	M8	2														0.69			

The numbers in the body of the table indicate the proportion of the total demand of parts whose operations are performed in the corresponding cell.

* Parts appearing in more than one column of this table represent lot splitting

Table 12. Part-Cell Assignment for Period 2

	Machine		Parts Types																
Cell	Type	Qnt.	3	11	12	16*	20	21	6*	7	13*	15	16*	18	2	4	6*	13*	25
C1	M1	1		1			0.94												
	M2	6	1	1	1		0.94												
	M3	2		1															
	M6	4	1		1		1	1											
	M7	1			1	0.13		1											
	M8	1				0.13		1											
C2	M1	3							0.18	1		1		1					
	M3	2								1		1		1					
	M4	3								1		1		1					
	M5	2							0.18	1	0.66								
	M7	3						0.60			0.66		0.87						
	M8	2									0.66	1	0.87						
	M9	1						0.60	0.18										
C3	M2	2					0.06										0.82		
	M5	1														1	0.82	0.34	
	M7	3						0.40							1			0.34	1
	M8	3													1			0.34	1
	M9	3						0.40							1	1	0.82		1
	M10	1													1	1			

Numbers outside the diagonal blocks indicate the presence of inter-cell movement.

requirement by letting $q = 0$. The resulting workload distributions and the corresponding objective function values are in Table 14. As it can be seen from this table, there are significant workload differences among the cells. In period 1, the workload difference between cell 1 and cell 3 is 569,967 processing time in minutes. If we assume the average processing time of the operations to be 12 minutes, then 47,498 more operations are performed in cell 1 than in cell 3. In period 2, cell 3 receives only half of the load of cell 1. Such unbalanced workload may lead to a poor performance of the system in terms of production throughput and increased work-in-process inventory. For $q = 0.9$, the workload is evenly distributed among the cells with 0.5% increment of the objective function value.

Table 13. Sample Values of $\eta_{jikl}(t)$

Period	Part No.	Cell	Machine, $\eta_{jikl}(t)$ Operation 1	2	3	4	5	6	7	8
1	15	1	1, 1.00	3, 1.00	4, 1.00	1, 1.00	4, 1.00	8, 1.00		
1	1	1	1, 1.00	3, 1.00	1, 1.00	3, 0.90 4, 0.10	4, 1.00			
1	10	1	5, 0.33	5, 0.33	5, 0.33	5, 0.33	9, 0.33	9, 0.33		
		2	5, 0.67	5, 0.67	5, 0.67	5, 0.67	9, 0.67	9, 0.67		
2	6	2	5, 0.18	5, 0.18	5, 0.18	1, 0.18	1, 0.18	9, 0.18		
		3	5, 0.82	5, 0.82	5, 0.82	2, 0.82	2, 0.82	9, 0.82		
2	21	2	9, 0.60	9, 0.60	7, 0.60	7, 060				
		3	9, 0.40	9, 0.40	7, 0.40	7, 040	7, 0.40			
		1					7, 0.60	6, 1.00	8, 1.00	8, 1.00

Table 14. Workload Distribution

q	Cell	Workload of Cells in Processing Time and as a Percentage of the Total Workload: Period 1		Period 2		Objective Function Value
0	1	1,527,821	40%	2,053,727	44%	2,613,864.00
	2	1,380,987	36%	1,597,487	34%	
	3	957,854	25%	1,025,005	22%	
0.9	1	1,259,986	33%	1,644,312	35%	2,626,995.00
	2	1,160,890	30%	1,616,507	35%	
	3	1,448,759	37%	1,397,494	30%	

Cost Savings: Cost savings may come from dynamic cell reconfiguration, lot splitting, and routing flexibility. To investigate the cost saving as a result of these features, we solved the model by eliminating these features one at a time. If we add the constraint:

$$y_{kl}^{-}(t) = y_{kl}^{+}(t) = 0, t \geq 2 \tag{25}$$

to the basic model, we can enforce that all required machines be installed at the beginning of period 1 and no system reconfiguration afterwards. If $Z_i(t)$ is set to 1 in (5), $\forall i, t$, then no lot-splitting can take place. If we add the constraint:

$$\sum_{t=1}^{T}\sum_{l=1}^{L}\eta_{1,1,1,l}(t) = 0 \tag{26}$$

to the model, the use of machine type 1 for the first operation on part 1 is prevented since this machine has a higher setup and operation cost than machine type 2, i.e., $[\frac{\mu_{1,1,1}}{B_1} + O_1 \times h_{1,1,1}] > [\frac{\mu_{1,1,2}}{B_1} + O_2 \times h_{1,1,2}]$. Similar constraints can be added corresponding to alternative routes. Thus each operation will have exactly one route and alternative routes will no longer be used in the cell formation decision. Adding the following constraints to the basic model, the model will select either a type 1 machine or a type 2 machine to process the first operation of part 1.

$$\sum_{l=1}^{L}\eta_{1,1,1,l}(t) \leq M \cdot X_1(t) \tag{27}$$

$$\sum_{l=1}^{L}\eta_{1,1,2,l}(t) \leq M \cdot (1 - X_1(t)) \tag{28}$$

$$X_1(t) \text{ is binary.} \tag{29}$$

Similar sets of constraints can be added for other parts and operations having alternative routings. This will prevent the coexistence of alternative routings.

Table 15. Cost Saving as a Result of Some Features of the Model

Feature inhibited from the basic model	Objective Function Value	Cost Saving by Considering the Feature	
None	2,626,995.00	NA	NA
Dynamic Reconfiguration of cells	2,661,179.00	34,184.00	1.3%
Lot Splitting	2,696,860.00	69,865.00	2.7%
Alternative Routings	2,732,050.00	105,055.00	4.0%
Coexistence of Alternative Routings	2,656,860.00	29,865.00	1.1%

By eliminating the features mentioned above one at a time from the basic model using the corresponding sets of constraints, we recalculated the example problem to observe the impacts on the solution of the model. The results are summarized in Table 15 and cost savings are significant for this example problem if dynamic reconfiguration, lot splitting, and routing flexibility are allowed.

3.2 Other Example Problems

We further illustrate the proposed model using ten other numerical examples, problems 2 to 11. The data for these examples were generated by varying the data related to the following problem aspects. Such variations are within relatively small ranges but do not follow any particular pattern.

- Number of planning periods and demands for part processing
- Number of cells, number of machines, machine capacities, machine procurement, holding, and operation costs
- Number of part types
- Numbers and sequences of operations of the parts
- Setup costs and processing times of the operations

General features of these additional problems are in Table 16. Complete data sets can be obtained from the authors upon request.

A summary of the impact of the workload balancing constraint on the workload distributions and objective function values of these 10 problems is in Table 17. In the third and fourth columns of this table are the maximum workload differences, expressed as a percentage of the total workloads. From these columns it can be seen that there are considerable workload differences in these examples if the workload balancing constraint is not imposed. A maximum workload difference of 37% is observed in problem 2, where one of the cells carries 55% and another cell carries only 18% of the total workload. The averages of the maximum workload differences of all the problems are 7.0% and 23.8% with and without the workload balancing constraint, respectively. The last column of this table are the increments of the objective function value due to workload balancing constraint. As can be seen from this column, the increment of the objective function value is less than 0.1% for the seven of the ten problems and the average percentage increment is 0.14%. In Table 18 we present the cost savings observed from these 10 example problems as a result of dynamic reconfiguration, lot splitting, alternative routings and allowing alternative routings to coexist. As can be seen from this table, lot splitting and alternative routing have resulted in significant cost savings with the averages being 5.95% and 6.47%, respectively. Cost saving from lot splitting can be due to reduced inter-cell movement, reduced machine investment, and better machine utilization. It can also enable workload balancing with minimal inter-cell movement since the processing of an operation of a batch can be allocated to different cells. The cost saving from alternative routings can be from reduced inter-cell movement, operation cost, setup costs, and machine investment cost since it can increase the number of ways in which the cells can be formed to reduce these costs. Dynamic reconfiguration and allowing alternative routings to coexist have resulted in considerable cost savings in these 10 problems with the averages being 0.50% and 0.53% respectively.

4. DISCUSSION AND CONCLUSION

In this paper, a comprehensive mathematical programming model for CMS design is proposed. A commercially available optimization software is used to solve the formulation for small size problems. Computational experience on such small problems showed that a significant amount of cost savings can be achieved by considering system reconfigurations, lot splitting and system flexibility. Our computational results also show that there are significant differences on workload distribution among the cells, if workload balancing is not attempted. Thus, with this work, we have demonstrated the importance of addressing several design issues in an integrated manner. Since

Table 16. Generic Attributes of the 10 Additional Problems

Problem No.	No of Planning Periods	No of Cells	No of Machines Types	No of Part Types	No of potentially non zero variables Integer	Total	No of Constraints
2	2	3	10	25	2120	6392	5694
3	2	3	10	25	2024	6098	5676
4	2	3	6	25	2040	6190	5376
5	2	3	6	25	1932	5860	5420
6	2	3	6	15	1392	4186	3664
7	2	3	6	15	1392	4186	3772
8	3	3	6	15	2088	6275	5506
9	3	3	6	15	2088	6284	5579
10	2	4	8	20	1856	5474	4928
11	2	4	8	20	5474	1856	4993

Table 17. Impacts of the Workload Balancing Constraint on the Workload Distributions and Objective Function Values of the 10 Arbitrarily Generated Problems

Problem No.	Maximum Cell Load Difference as % of Total: Period	$q = 0$	$q = 0.9$	Objective Value Increment %
2	1	37	10	0.05
	2	29	8	
3	1	25	3	0.11
	2	24	10	
4	1	33	4	0.03
	2	26	4	
5	1	31	7	0.01
	2	35	10	
6	1	11	6	0.02
	2	29	7	
7	1	33	9	0.04
	2	18	6	
8	1	10	9	0.07
	2	27	8	
	3	9	8	
9	1	13	10	0.33
	2	11	6	
	3	32	10	
10	1	21	7	0.04
	2	15	5	
11	1	29	4	0.74
	2	25	4	
Average	–	23.8	7.0	0.14

the proposed mixed integer programming model is NP-hard, we are currently developing heuristic methods to efficiently solve the proposed model for problems of larger sizes. The heuristic methods will be designed to generate several near optimal alternative solutions that will be further evaluated for their performances related to machine utilization, work-in-process inventory, and due date.

Acknowledgements: This research is supported by Discovery Grant from NSERC of Canada and by Faculty Research Support Fund from the Faculty of Engineering and Computer Science, Concordia University, Montreal, Quebec, Canada. The authors sincerely thank the three anonymous referees for their thorough reviews of the early versions of this paper and their very valuable comments.

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Table 18. Cost Savings Resulted from Some Features of the Model on 10 Arbitrarily Generated Problems

Problem No.	Feature Eliminated-Objective/Cost Saving: None	Dynamic Reconfiguration	Lot Splitting	Alternative Routing	Coexistence of Alternative Routings
2	3,296,888.00	3,308,990.00	3,661,735.00	3,364,681.00	3,326,555.00
	NA	12,102.00	364,847.00	67,793.00	29,667.00
3	3,794,331.00	3,804,331.00	3,879,594.00	3,848,139.00	3,824,084.00
	NA	10,000.00	85,263.00	53,808.00	29,753.00
4	2,022,009.00	2,024,922.00	2,065,277.00	2,088,094.00	2,023,695.00
	NA	2,913.00	43,268.00	66,085.00	1,686.00
5	2,924,824.00	2,929,882.00	2,986,842.00	3,072,987.00	2,940,365.00
	NA	5,058.00	62,018.00	148,163.00	15,541.00
6	1,529,347.00	1,548,218.00	1,548,005.00	1,666,412.00	1,530,340.00
	NA	18,871.00	18,658.00	137,065.00	993.00
7	1,535,497.00	1,538,605.00	1,549,114.00	1,705,372.00	1,536,560.00
	NA	3,108.00	13,617.00	169,875.00	1,063.00
8	2,067,705.00	2,070,649.00	2,141,568.00	2,283,562.00	2,069,057.00
	NA	2,944.00	73,863.00	215,857.00	1,352.00
9	2,214,939.00	2,227,884.00	2,556,638.00	2,246,319.00	2,231,734.00
	NA	12,945.00	341,699.00	31,380.00	16,795.00
10	1,744,313.00	1,757,518.00	1,844,063.00		
	NA	13,205.00	99,750.00		
11	2,439,456.00	2,448,741.00	2,511,847.00	2,478,229.00	2,449,698.00
	NA	9,285.00	72,391.00	38,773.00	10,242.00
Average Saving in Percent		0.50	5.95	6.47	0.53

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